UNIT 5

1. $\frac{\csc\theta}{\sec\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\frac{1}{\sin\theta}}{\frac{1}{\cos\theta}} + \frac{\cos\theta}{\sin\theta} = \frac{\cos\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} = \cot\theta + \cot\theta = 2\cot\theta$

2. $\frac{\sec^2 x - \tan^2 x + \tan x}{\sec x} = \frac{1+\tan x}{\sec x} = \frac{1}{\sec x} + \frac{\tan x}{\sec x} = \cos x + \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x}} = \cos x + \sin x$

3. $1 - \frac{\sin^2\theta}{1-\cos\theta} = 1 - \frac{1-\cos^2 x}{1-\cos x} = 1 - \frac{(1+\cos x)\cancel{(1-\cos x)}}{\cancel{1-\cos x}} = 1-(1+\cos x) = -\cos x$

4. $\frac{\tan x}{1+\sec x} + \frac{1+\sec x}{\tan x} = \frac{\tan^2 x}{\tan x(1+\sec x)} + \frac{(1+\sec x)^2}{\tan x(1+\sec x)} = \frac{\tan^2 x + 1 + 2\sec x + \sec^2 x}{\tan x(1+\sec x)} = \frac{\sec^2 x + 2\sec x + \sec^2 x}{\tan x(1+\sec x)}$

$= \frac{2\sec^2 x + 2\sec x}{\tan x(1+\sec x)} = \frac{2\sec x(\sec x + 1)}{\tan x(1+\sec x)} = \frac{2\sec x}{\tan x} = \frac{2\left(\frac{1}{\cos x}\right)}{\left(\frac{\sin x}{\cos x}\right)} = \frac{2}{\sin x} = 2\csc x$

5. $\frac{\cos\left(\frac{\pi}{2}-x\right)}{1-\cos x} = \frac{\sin x}{1-\cos x} \cdot \frac{1+\cos x}{1+\cos x} = \frac{\sin x(1+\cos x)}{1-\cos^2 x} = \frac{\sin x(1+\cos x)}{\sin^2 x} = \frac{1+\cos x}{\sin x} \cdot \frac{\left(\frac{1}{\cos x}\right)}{\left(\frac{1}{\cos x}\right)} = \frac{\frac{1}{\cos x} + \frac{\cos x}{\cos x}}{\frac{\sin x}{\cos x}}$

$= \frac{\sec x + 1}{\tan x}$

6. $\frac{\cos x}{1+\sin(-x)} = \frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x(1+\sin x)}{1-\sin^2 x} = \frac{\cos x(1+\sin x)}{\cos^2 x} = \frac{1+\sin x}{\cos x} = \frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$= \sec x + \tan x$

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1. $\cot^2 x - \cot^2 x\cos^2 x = \cot^2 x(1-\cos^2 x) = \cot^2 x(\sin^2 x) = \frac{\cos^2 x}{\sin^2 x} \cdot \sin^2 x = \cos^2 x$

2. $(\sec^3\theta - \sec^2\theta)(-\sec\theta + 1) = \sec^2\theta(\sec\theta - 1) - 1(\sec\theta - 1) = (\sec^2\theta - 1)(\sec\theta - 1) = \tan^2\theta(\sec\theta - 1)$

3. $\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = \frac{1-\cos\theta}{1-\cos^2\theta} + \frac{1+\cos\theta}{1-\cos^2\theta} = \frac{2}{1-\cos^2\theta} = \frac{2}{\sin^2\theta} = 2\csc^2\theta$

4. $\tan x - \frac{\sec^2 x}{\tan x} = \frac{\tan^2 x}{\tan x} - \frac{\sec^2 x}{\tan x} = \frac{\tan^2 x - \sec^2 x}{\tan x} = \frac{-1}{\tan x} = -\cot x$

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1. $\tan\theta = \sqrt{3}$

$\theta = \frac{\pi}{3}, \frac{4\pi}{3}$

2. $\csc\theta = \sqrt{2}$

$\sin\theta = \frac{\sqrt{2}}{2}$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

3. $2\tan^2\theta = 2$

$\tan^2\theta = 1$

$\tan\theta = \pm 1$

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

4. $-4\cos\theta = 2\sqrt{3}$

$\cos\theta = \frac{-\sqrt{3}}{2}$

$\theta = \frac{5\pi}{6}, \frac{7\pi}{6}$

5. $-1 + 8\sin\theta = -4\sqrt{3} - 1$

$8\sin\theta = -4\sqrt{3}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = \frac{4\pi}{3}, \frac{5\pi}{3}$

6. $\sin^2 x - \sin x - 2 = 0$

$(\sin x - 2)(\sin x + 1) = 0$

$\sin x = 2$ N.S. $\quad \sin x = -1$

$x = \frac{3\pi}{2}$

7. $\sec^2 x - \sec x - 2 = 0$

$(\sec x - 2)(\sec x + 1) = 0$

$\sec x = 2 \quad \sec x = -1$

$\cos x = \frac{1}{2} \quad \cos x = -1$

$x = \frac{\pi}{3}, \frac{5\pi}{3}, \pi$

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1. $\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45 \cos 30 - \sin 30 \cos 45$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

2. $\cos\frac{11\pi}{12} = \cos\left(\frac{2\pi}{3} + \frac{\pi}{4}\right)$

$= \cos\frac{2\pi}{3}\cos\frac{\pi}{4} - \sin\frac{2\pi}{3}\sin\frac{\pi}{4}$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{-\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \frac{-\sqrt{2} - \sqrt{6}}{4}$

3. $\tan 165^\circ = \tan(120^\circ + 45^\circ)$

$= \frac{\tan 120 + \tan 45}{1 - \tan 120 \tan 45}$

$= \frac{-\sqrt{3} + 1}{1 - (-\sqrt{3})(1)}$

$= \frac{1 - \sqrt{3}}{1 + \sqrt{3}} \cdot \frac{1 - \sqrt{3}}{1 - \sqrt{3}}$

$= \frac{1 - 2\sqrt{3} + 3}{1 - 3}$

$= \frac{4 - 2\sqrt{3}}{-2}$

$= -2 + \sqrt{3}$

4. $\csc 75^\circ = \frac{1}{\sin 75^\circ}$

$= \frac{4}{\sqrt{2} + \sqrt{6}}$

$= \frac{4}{\sqrt{2} + \sqrt{6}} \cdot \frac{\sqrt{2} - \sqrt{6}}{\sqrt{2} - \sqrt{6}}$

$= \frac{4(\sqrt{2} - \sqrt{6})}{2 - 6}$

$= \frac{4(\sqrt{2} - \sqrt{6})}{-4}$

$= -1(\sqrt{2} - \sqrt{6})$

$= \sqrt{6} - \sqrt{2}$

So, $\sin 75^\circ = \sin(30 + 45)$

$= \sin 30 \cos 45 + \sin 45 \cos 30$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2} + \sqrt{6}}{4}$

5. $\sin 2x \cos 5x - \sin 5x \cos 2x$

$= \sin(2x - 5x)$

$= \sin(-3x)$

$= -\sin 3x$

6. $\dfrac{\tan(x+1) + \tan(x-1)}{1 - \tan(x+1)\tan(x-1)}$

$= \tan((x+1)+(x-1))$

$= \tan(2x)$

7. $\cos 221^\circ \cos 79^\circ - \sin 221^\circ \sin 79^\circ$

$= \cos(221^\circ + 79^\circ)$

$= \cos 300^\circ$

$= \frac{1}{2}$

8. $\dfrac{\tan 586^\circ - \tan 466^\circ}{1 + \tan 586^\circ \tan 466^\circ}$

$= \tan(586^\circ - 466^\circ)$

$= \tan 120^\circ$

$= -\sqrt{3}$

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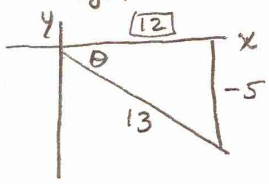
9. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{5}{13}\right)\left(\frac{12}{13}\right)$

$= \frac{-120}{169}$

sin is negative
tan is negative } Q4



$360 \div 2 = 180$

$270 \div 2 = 135$

10. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{12}{13}\right)^2 - \left(-\frac{5}{13}\right)^2$

$= \frac{144}{169} - \frac{25}{169}$

$= \frac{119}{169}$

11. $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

In Q2

$= +\sqrt{\dfrac{1 - \frac{12}{13}}{2}}$

$= \sqrt{\dfrac{\frac{13}{13} - \frac{12}{13}}{2}}$

$= \sqrt{\dfrac{\frac{1}{13}}{2}}$

$= \sqrt{\frac{1}{26}} = \frac{1}{\sqrt{26}} = \frac{\sqrt{26}}{26}$

12. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$ or $\dfrac{\sin 2\theta}{\cos 2\theta}$

$= \dfrac{\left(\frac{-120}{169}\right)}{\left(\frac{119}{169}\right)}$

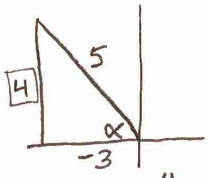
$= -\frac{120}{119}$

13. $\sin(\beta-\alpha) = \sin\beta\cos\alpha - \sin\alpha\cos\beta$

$= \left(\frac{-8}{17}\right)\left(\frac{-3}{5}\right) - \left(\frac{4}{5}\right)\left(\frac{-15}{17}\right)$

$= \frac{24}{85} + \frac{60}{85}$

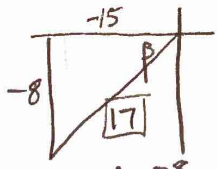
$= \frac{84}{85}$



$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{-3}{5}$

$\tan\alpha = -\frac{4}{3}$



$\sin\beta = \frac{-8}{17}$

$\cos\beta = \frac{-15}{17}$

$\tan\beta = \frac{8}{15}$

14. $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(-\frac{3}{5}\right)\left(\frac{-15}{17}\right) + \left(\frac{4}{5}\right)\left(\frac{-8}{17}\right)$

$= \frac{45}{85} - \frac{32}{85}$

$= \frac{13}{85}$

15. $\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$= \frac{\left(-\frac{4}{3}\right) + \left(\frac{8}{15}\right)}{1 - \left(\frac{-4}{3}\right)\left(\frac{8}{15}\right)}$

$= \frac{\left(-\frac{4}{5}\right)}{\left(\frac{77}{45}\right)}$

$= -\frac{36}{77}$

16. $2\cos(4x) + 1 = 0$

$\cos(4x) = -\frac{1}{2}$

$4x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Set 1: $x = \frac{\pi}{6}, \frac{\pi}{3}$

(+ Period)

Set 2: $x = \frac{2\pi}{3}, \frac{5\pi}{6}$

Set 3: $x = \frac{7\pi}{6}, \frac{4\pi}{3}$

Set 4: $x = \frac{5\pi}{3}, \frac{11\pi}{6}$

17. $4\sin(2x) - 3 = 0$

$\sin(2x) = \frac{3}{4}$

$2x = \sin^{-1}\left(\frac{3}{4}\right) = 48.59° \text{ \& } 131.41°$

Set 1: $x = 24.295°, 65.705°$

(Add Period)

Set 2 $x = 204.295°, 245.705°$

In Radians: $x = 0.135\pi, 0.365\pi, 1.135\pi, 1.365\pi$

18. $\tan\left(\frac{2x}{3}\right) - 1 = 0$

$\tan\left(\frac{2x}{3}\right) = 1$

$\frac{2x}{3} = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = \frac{3\pi}{8}, \frac{15\pi}{8}$

19. $27 - 2\tan x = 0$

$-2\tan x = -27$

$\tan x = 13.5 \leftarrow$ Q1, Q3

$x = \tan^{-1}(13.5)$

Deg: $x = 85.76°, 265.76°$

Rad: $x = 1.497$ RAD, 4.638 RAD

20. $2\tan^2(2x) - 6 = 0$

$\tan^2(2x) = 3$

$\tan(2x) = \pm\sqrt{3}$

$2x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

Set 1 : $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{5\pi}{6}$

Set 2 : $x = \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{11\pi}{6}$

21. $4\sin^2\left(\frac{x}{2}\right) - 3 = 0$

$\sin^2\left(\frac{x}{2}\right) = \frac{3}{4}$

$\sin\left(\frac{x}{2}\right) = \pm\frac{\sqrt{3}}{2}$

$\frac{x}{2} = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $\cancel{\frac{8\pi}{3}}, \cancel{\frac{10\pi}{3}}$

UNIT 6

1. $c^2 = a^2 + b^2 - 2ab\cos C$

$13^2 = 9^2 + 8^2 - 2(9)(8)\cos C$

$169 = 145 - 144\cos C$

$24 = -144\cos C$

$\frac{-24}{144} = \cos C$

$C = \cos^{-1}\left(\frac{-24}{144}\right) = 99.59°$

2. $\measuredangle C = 180 - 25 - 75 = 80°$

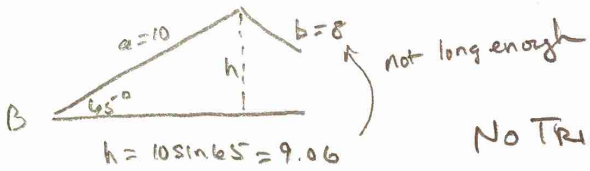
$\frac{\sin 80}{12} = \frac{\sin 75}{b}$

$b = \frac{12\sin 75}{\sin 80} = 11.77$

3. $\frac{\sin 62°}{14} = \frac{\sin 49°}{c}$

$c = \frac{14\sin 49°}{\sin 62°} = 11.97$

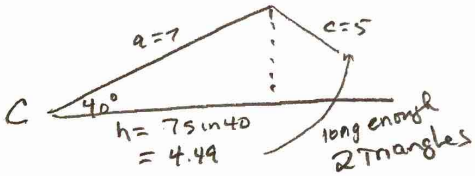
4. Ambiguous Case



$h = 10\sin 65 = 9.06$

NO TRIANGLE

5. Ambiguous Case



$h = 7\sin 40 = 4.49$

TRIANGLE 1

$A_1 = 64.15°$ $a = 7$

$B_1 = 75.85°$ $b = 7.54$

$C = 40°$ $c = 5$

Supplement

TRIANGLE 2

$A_2 = 115.85$ $a = 7$

$B_2 = 24.15°$ $b = 3.18$

$C = 40°$ $c = 5$

$\frac{\sin A}{7} = \frac{\sin 40}{5}$

$\sin A = \frac{7\sin 40}{5}$

$A = \sin^{-1}(\checkmark) = 64.15$

$B = 180 - A - C$

$B_1 = 75.85°$

$B_2 = 24.15°$

$\frac{\sin 40}{5} = \frac{\sin 75.85}{b_1}$

$b_1 = \frac{5\sin 75.85}{\sin 40}$

$b_1 = 7.54$

$\frac{\sin 40}{5} = \frac{\sin 24.15}{b_2}$

$b_2 = \frac{5\sin 24.15}{\sin 40}$

$b_2 = 3.18$

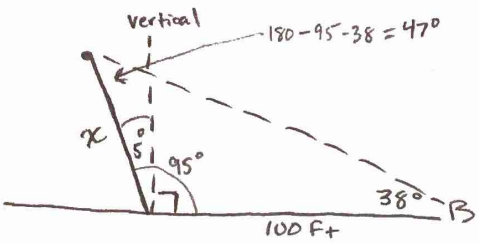
6. $\text{Area} = \frac{1}{2}(15)(17)\sin 103°$

$= 124.23 \text{ units}^2$

7. $\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{1067.5(1067.5-675)(1067.5-525)(1067.5-935)} = 173{,}544.791 \text{ ft}^2$

$s = \dfrac{675+525+935}{2} = 1067.5$

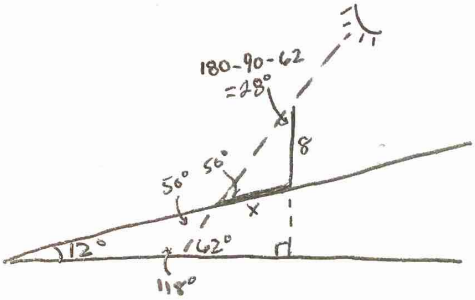
8.



$\dfrac{\sin 47^\circ}{100} = \dfrac{\sin 38^\circ}{x}$

$x = \dfrac{100 \sin 38^\circ}{\sin 47^\circ} = 84.18 \text{ ft}$

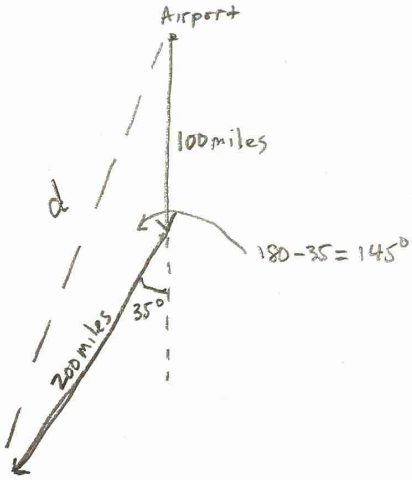
9.



$\dfrac{\sin 50^\circ}{8} = \dfrac{\sin 28^\circ}{x}$

$x = \dfrac{8 \sin 28^\circ}{\sin 50^\circ} = 4.9 \text{ ft}$

10.



SAS

$d^2 = 100^2 + 200^2 - 2(100)(200)\cos 145^\circ$

$d^2 = 50{,}000 - 40{,}000 \cos 145$

$d^2 = 50{,}000 - (-32766.08177)$

$d^2 = 82{,}766.08177$

$d = 287.69 \text{ miles}$